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COMPARISON BETWEEN THEORY AND EXPERIMENT OF NONLINEAR PROPAGATION FOR 4.5-CYCLE OPTICAL PULSES IN A FUSED-SILICA FIBER

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Abstract A nonlinear fiber equation that can describe optical-pulse propagation in the single-cycle regime is derived. An equation that includes self-phase modulation, the self-steepening effect and the Raman effect is shown. In the numericdal calculations, all orders of the linear dispersion terms are included. In the experiment, 4.5-cycle optical pulses are propagated in a single-mode fused-silica fiber and the broadband spectrum is generated by dispersive self-phase modulation. Calculated spectra are compared with the experimental spectrum. It is found that the inclusion of the dispersion of the effective core area and the Raman effect is important to correctly describe the observed spectrum.

INTRODUCTION

Conventionally, the slowly-varying envelope approximation (SVEA) has been used to describe propagation of an optical pulse in a fiber.[1] However, if the pulse duration becomes close to the optical cycle time, this approximation becomes invalid. Also, its treatment of linear dispersion of the fiber, i.e., including only up to 3rd-order terms around the center frequency of the pulse, becomes questionable when the bandwidth of the pulse becomes extremely large. Recently, Brabec and Krausz theoretically showed[2] that the envelope description of the electric field can be still used for the pulse whose duration is as short as one optical cycle under some conditions. Their numerical calculations include the steepening term, dispersion terms up to 2nd-order terms, and the instantaneous self-phase modulation (SPM) term. Following their derivation except for the following points, we derived an equation for fiber propagation. That is, unlike their derivation, our method includes the Raman effect, *all orders* of terms for linear material and waveguide dispersion, and the dispersion of the effective core area. Thus our equation can be also used to describe propagation in a long distance where the dispersion effect becomes important.

To check the validity of equations thus derived, we calculated the spectrum broadening by dispersive self-phase modulation (SPM) in a single-mode fused-silica fiber with the input pulse whose duration is 12 fs (4.5-cycle). The calculated spectra are compared with the experimental spectrum. Equations including the Raman effect and the dispersion of the effective core area were derived by Blow and Wood[3] and Mamyshev and Chernikov.[4] However no explicit method to include all orders of linear dispersion terms was shown and no comparisons between calculated and experimental results for several cycle pulses like here were performed.

DERIVATION OF EQUATIONS

We start from the Fourier-transformed Maxwell equation:

$$(\nabla_\perp^2 + \partial_z^2 + \frac{\varepsilon(\omega)\omega^2}{c^2})\tilde{\mathbf{E}} = -\mu_0\omega^2\tilde{\mathbf{P}}_{\mathbf{NL}},$$

where $\tilde{\mathbf{E}} = \int_{-\infty}^{\infty} \mathbf{E}\exp(i\omega t)dt$, $\tilde{\mathbf{P}}_{\mathbf{NL}} = \int_{-\infty}^{\infty} \mathbf{P}_{\mathbf{NL}}\exp(i\omega t)dt$, $\mathbf{E}$ is the electric field, $\mathbf{P}_{\mathbf{NL}}$ is the nonlinear polarization, c is the speed of light, μ_0 is the vacuum permeability, $\varepsilon(\omega)$ is the linear dielectric constant, ω is the angular frequency and $\nabla_\perp^2 = \partial_x^2 + \partial_y^2$. We consider the propagation in z direction and assume that all the fields are polarized in one direction and consider only that component. We also assume that the electric field and the nonlinear polarization envelope functions can be written in the following form:

$$\begin{aligned}\tilde{E}(x,y,z,\omega) &= F(x,y,\omega)\tilde{A}(z,\omega-\omega_0)\exp(i\beta_0 z),\\ \tilde{P}_{NL}(x,y,z,\omega) &= \varepsilon_0\chi^{(3)}F(x,y,\omega)^3\tilde{p}_{NL}(z,\omega-\omega_0)\exp(i\beta_0 z),\end{aligned}$$

where ε_0 is the vacuum dielectric constant and β_0 is the real part of the propagation constant at the center angular frequency ω_0. Here, we consider only the third-order nonlinear optical effect with the coefficient $\chi^{(3)}$. We assume that we can separate the forward propagating wave and the backward propagating wave and consider only the forward propagating one. Using the first-order perturbation theory,[1] we obtain

$$(\partial_z^2 + 2i\beta_0\partial_z - \beta_0^2 + \gamma(\omega)^2)\tilde{A}(z,\omega-\omega_0) = -\frac{\omega^2 N(\omega)\chi^{(3)}}{c^2}\tilde{p}_{NL}(z,\omega-\omega_0),$$

where $\gamma(\omega) \equiv \beta(\omega) + i\alpha(\omega)/2$ is the propagation constant without the nonlinear term and is determined by the unperturbed cross-sectional field equation, $(\nabla_\perp^2 + \varepsilon\omega^2/c^2)F = \gamma^2 F$. Also, $N(\omega) = \int F^4 dxdy / \int F^2 dxdy$. Thus, $\gamma(\omega)$ contains both the material dispersion and the waveguide dispersion. Converting the unit of $\tilde{A}$ to [watt$^{1/2}$] by multiplying $(2/\varepsilon_0 cn)^{1/2}/(\int F^2 dxdy)^{1/2}$ and using the relations $\chi^{(3)} =$

$8nn_2/3$ and $n_2^I = 2n_2/\varepsilon_0 cn$, where n is the linear index of refraction of the medium, we have

$$(\partial_z^2 + 2i\beta_0\partial_z - \beta_0^2 + \gamma^2)\tilde{A}(z,\omega-\omega_0) = -\frac{8nn_2^I\omega^2}{3c^2A_{\rm eff}(\omega)}\tilde{p}_{NL}(z,\omega-\omega_0). \quad (1)$$

Here, $A_{\rm eff}(\omega) = (\int F^2dxdy)^2/\int F^4dxdy$ is the effective core area. By inverse Fourier transforming Equation (1), converting the time coordinate such that the pulse center is always at its origin as $T = t - \dot{\beta}_0 z$ ($\dot{\beta}_0 = \partial_\omega(\beta)_{\omega_0}$) and $\xi = z$ ($\partial_t = \partial_T, \partial_z = \partial_\xi - \dot{\beta}_0\partial_T$), and operating $(2i\beta_0(1+i\dot{\beta}_0\partial_T/\beta_0))^{-1}$ to both sides of equation, we have

$$\left[\frac{1}{2i\beta_0}(1+i\frac{\dot{\beta}_0}{\beta_0}\partial_T)^{-1}(\partial_\xi^2+\widehat{D}'^2)+\partial_\xi - i\widehat{D}'\right]A(\xi,T) =$$
$$i\frac{4g(\omega_0)\omega_0^2}{3c^2\beta_0}(1+i\frac{\dot{\beta}_0}{\beta_0}\partial_T)^{-1}(1+\frac{i}{\omega_0}\partial_T)^2(1+i\partial_\omega \ln g|_{\omega_0}\partial_T)p_{NL}(\xi,T), \quad (2)$$

where $g(\omega) = n(\omega)n_2^I(\omega)/A_{\rm eff}(\omega)$ and we include the dispersion of $g(\omega)$ up to the first order terms in the Taylor expansion at ω_0, and $\widehat{D}' = \sum_{n=0}^{\infty}\frac{i^n}{n!}\partial_\omega^n(\beta+\frac{i\alpha}{2})_{\omega_0}\partial_t^n - \beta_0 - i\dot{\beta}_0\partial_T$. The first term of the left hand side of this equation can be ignored if $|\partial_\xi A| \ll \beta_0|A|$ which physically means that the spatial variation of the envelope is much larger than the wavelength. Also, we can show that

$$(1+i\frac{\dot{\beta}_0}{\beta_0}\partial_T)^{-1}(1+\frac{i}{\omega_0}\partial_T)^2 = 1+i(\frac{2}{\omega_0}-\frac{\dot{\beta}_0}{\beta_0})\partial_T - (\frac{\dot{\beta}_0}{\beta_0}-\frac{1}{\omega_0})^2\partial_T^2(1+i\frac{\dot{\beta}_0}{\beta_0}\partial_T)^{-1}.$$

The third term of the right hand side of the above equation can be neglected if the difference between the group velocity ($v_g = 1/\dot{\beta}_0$) and the phase velocity ($v_p = \omega_0/\beta_0$) of the pulse is small. These two approximations are the same as the slowly-evolving-wave approximation (SEWA) in ref. 2, where there are no conditions specifying the slowness of the temporal change of the envelope compared with the optical cycle time. Thus this equation can be used for the pulse as short as single optical cycle. By using these approximations, Equation (2) becomes

$$\partial_\xi A(\xi,T) = i(\widehat{D}'+\widehat{D}_{\rm corr})A(\xi,T) + i\frac{4g(\omega_0)\omega_0^2}{3c^2\beta_0}(1+is\partial_T)p_{NL}(\xi,T), \quad (3)$$

where $\widehat{D}_{\rm corr} = (1+i\dot{\beta}_0\partial_T/\beta_0)^{-1}\widehat{D}'^2/2\beta_0$ and $s = 2/\omega_0 - \dot{\beta}_0/\beta_0 + \partial_\omega \ln g|_{\omega_0}$.

Nonlinear term

The nonlinear term p_{NL} may contain both the instantaneous Kerr nonlinearity as well as the Raman response as follows

$$p_{NL}(\xi,T) = \frac{1}{4}\int_0^\infty R(T')[2|A(\xi,T-T')|^2A(\xi,T) + A^2(\xi,T-T')A^*(\xi,T)\exp(2i\omega_0T')]dT', \quad (4)$$

where the response function $R(t)$ is given by $R(T) = (1 - f_R)\delta(T) + f_R h_R(T)$. For the fused-silica, $f_R = 0.3$ and $h_R(T) = ((\tau_1^2 + \tau_2^2)/\tau_1\tau_2^2)\exp(-T/\tau_2)\sin(T/\tau_1)$ where $\tau_1 = 12.2$ fs and $\tau_2 = 32$ fs.[1] Since the change of h_R is much slower than the optical cycle time, Equation (4) can be written as

$$p_{NL}(\xi, T) = \frac{3}{4}[(1 - f_R)|A(\xi, T)|^2 + \frac{2}{3} f_R \int_0^\infty h_R(T')|A(\xi, T - T')|^2 dT']A(\xi, T). \quad (5)$$

Numerical calculations

Equation (3) can be calculated by the split-step Fourier method.[1] Dispersion terms can be evaluated in the frequency domain as:

$$\widehat{D}'(\omega) = \beta(\omega) + \frac{i\alpha(\omega)}{2} - \beta_0 - \dot{\beta}_0(\omega - \omega_0), \quad (6)$$

$$\widehat{D}_{corr}(\omega) = \frac{\widehat{D}'^2(\omega)}{2(\beta_0 + \dot{\beta}_0(\omega - \omega_0))}. \quad (7)$$

In the numerical calculations, the Sellmeier equation is used to model the dispersion relations of a fused-silica fiber.[1] In practice, limited numbers of frequency points used in the fast Fourier transform routine are used in calculations. At the beginning of the program, the values of Equations (6) and (7) are evaluated at each frequency point only the one time and they are stored in the memory. Thus calculation of this method is very efficient.

FUSED-SILICA FIBER EXPERIMENT

In the experiment, pulses from a Ti:Sapphire oscillator (Femtosource M-1, center wavelength 798 nm) were introduced in a 2.5 mm-long single-mode fused-silica fiber (Newport F-SPV, core radius 1.316 μm) by the dispersion-free reflective objective to prevent pulse-broadening. The pulse duration measured by the fringe-resolved autocorrelator was 12 fs corresponding to a 4.5-cycle pulse and the peak power of the input pulse obtained from the measured pulse energy at the fiber output was 175 kW. The temporal shape of the input pulse for the numerical calculations was obtained by the inverse Fourier-transform of the input pulse spectrum. However the transform-limited pulse duration (8.6 fs) was smaller than the experimentally measured pulse duration (12 fs) due to the residual chirp in the pulse. To account for this, the negative linear-chirp was assumed in the input pulse such that its width became 12 fs.

We considered only the material dispersion of the fused-silica fiber using the Sellmeier equation[1] with zero loss. The calculated second and third order dispersion coefficients were $\beta_2 = 3.65 \times 10^{-26}$ s^2/m and $\beta_3 = 2.76 \times 10^{-41}$ s^3/m at the center wavelength. For the single-mode silica fiber, it is well known that the effective core area depends on the wavelength and it increases rapidly as the wavelength becomes longer than the cut-off wavelength of the fiber.[1] For the fiber used in this experiment, the cut-off wavelength is 550 nm. In this case, the effective core area is $A_{\rm eff} = 2.055\pi a^2$ at the center wavelength 798 nm, but it is $A_{\rm eff} = 3.68\pi a^2$ at 1000 nm. Thus it is expected that at the wavelength longer than the center wavelength, the nonlinear effect becomes smaller since the effective core area becomes larger. To take into account this effect in the calculation, the term $\partial_\omega \ln g|_{\omega_0}$ in eq. (3) was calculated at the center wavelength and used as the additional steepening parameter. This value at the center wavelength was numerically calculated as $-\omega_0 \partial_\omega \ln A_{\rm eff} = 2.06$. For the nonlinear refractive index of the fused-silica fiber, we used the value $n_2^I = 2.48 \times 10^{-20}$ m^2/W from ref. 5.

RESULT

We calculated spectra with the following three methods: (case a); the rigorous linear dispersion terms and the steepening term including the dispersion of the effective core area (Equations (3) and (5)), (case b); same as case a without the Raman term ($f_R = 0$ in Equation (5)), and (case c); the SVEA (up to the 3rd-order dispersion terms without the steepening nor the Raman terms). In Figure 1, the experimental spectra as well as the calculated spectra are shown. It is seen that for the SVEA, the spectrum intensity at longer wavelength than the center wavelength is much larger than that in the experiment. The spectra of cases b and c are closer to that in the experiment, indicating that it is important to include the steepening term, especially from the dispersion of the effective core area. It is seen that by including the Raman term (case a), the spectrum width becomes smaller and the agreement between the experiment and the calculation becomes better. It is found that the calculated spectrum including the rigorous dispersion terms and that including only up to the 3rd-order terms are almost identical. Thus, in this case, the approximation using up to 3rd-order terms is sufficient.

CONCLUSION

The nonlinear pulse propagation equation that includes all orders of the linear dis-

persion terms, the steepening effect, dispersion of the effective core area and the Raman effect is derived. This can be used for pulses in the single-cycle regime. This is used in the calculations for SPM spectra using a single-mode fused-silica fiber and the calculated spectra are compared with the experimental spectrum obtained using a 4.5-cycle input pulse. It is found that the steepening term is quite large due to the dispersion of the effective core area and its inclusion is very important to reproduce the correct shape of the spectrum. Also it is found that by including the Raman term, the spectral broadening becomes smaller, which agrees better with the experimental spectra.

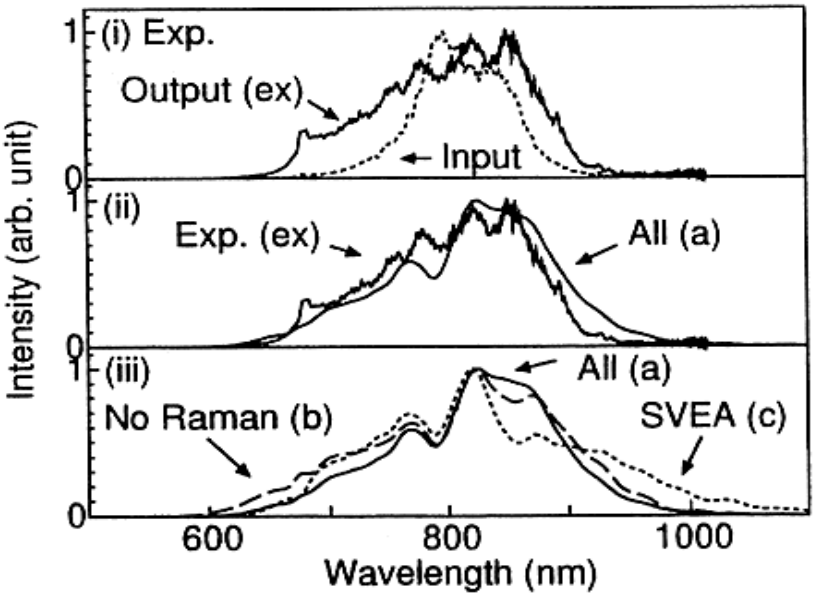

FIGURE 1 (i) Experimental SPM spectrum (ex) using a 2.5 mm-long fused-silica fiber. The input spectrum is shown by a dotted line. (ii) Comparison between experimental (ex) and calculated (a) spectra. Calculation includes the steepening term, the instantaneous SPM term, the Raman term, all terms of linear material dispersion, and the dispersion of the effective core area. (iii) Comparison between calculated spectra, (a) ; same as (ii), (b) ; same as (ii) except for that the Raman term is not included, (c) ; the SVEA is used.

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